(ii) the spectrometer does not allow determination directional values since the detection is realized according to a defined solid angle.

A more accurate model including exact scattering phenomena is actually studied.

CONCLUSION

We used Lee's and Kerker's theories to calculate the fibrous medium radiative properties for the case of fibres oriented in stratification planes parallel to the boundaries. The application of our results to a layer of silica fibres allowed us to determine the absorption, scattering and extinction coefficients and the backscatter factor. In addition, we determined a radiative conductivity from a two flux model showing the influence of the medium thickness and the fibre diameter. Finally, an experimental study in transmission allowed us to confirm our theoretical results, particularly concerning the existence of the two Christiansen filters; thickness effect and the optimal fibre diameter existence being already shown by measurements of radiative conductivity.

Despite this, a complementary study remains necessary to obviate the limits assigned by the two flux model and to obtain a more reliable characterization of the pure radiative transfer.

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Correlations of pressure drop in packed beds taking into account the effect of confining wall

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INTRODUCTION

THE ABILITY to predict reliably the pressure drop through packed beds is of great significance since pumping costs are directly related to the pressure drop of the system in question. When pressure drop and flow information is required for a wide range of Reynolds number in packed bed configurations, the Ergun equation [1] has been the favourite choice amongst literature correlations. Ergun's equation effectively accounts for simultaneous inertia and viscous energy losses, and a fair amount of pressure drop-flow data have been correlated by it for beds with various geometrically shaped particles. Macdonald *et al.* [2] gave a good account of the various published models, and divided them into roughly three categories: (a) phenomenological models; (b) models based on conduit flow: (i) geometrical models, (ii) statistical models and (iii) models utilizing the complete Navier–Stokes equation; (c) models based on flow around submerged objects. There is, however, a great deal of overlap between these models. The simplest pressure drop correlation is the one proposed by Ahmed and Sunada [3]

$$-\frac{\nabla P}{\mu V_0} = \alpha + \beta \frac{\rho V_0}{\mu} \quad (1)$$

where V_0 is the superficial velocity, μ and ρ are, respectively, the viscosity and density of the fluid, and ∇P the pressure gradient. α and β are model parameters to be established empirically by a least square fitting procedure. However, despite its attractive simplicity, the most serious drawback to equation (1) is the lack of parameters characterising the porous medium. It is for this reason that Macdonald *et al.* [2] proposed a modified form of Ergun equation (2), where the constants 150 and 1.75 were replaced, respectively, by A and B

$$\frac{\Delta P}{L} = \frac{150\mu u(1-\varepsilon_m)^2}{d_p^2\varepsilon_m^3} + \frac{1.75\rho u^2(1-\varepsilon_m)}{d_p\varepsilon_m^2} \quad (2)$$

rearranging equation (2) we obtain

$$\frac{\Delta P d_p^2\varepsilon_m^3}{L\mu u(1-\varepsilon_m)^2} = B\frac{\rho u d_p}{\mu(1-\varepsilon_m)} + A. \quad (3)$$

In equation (3), the left-hand side is called the modified friction factor, f', while the term $\rho u d_p/\mu(1-\varepsilon_m)$ is referred to as the modified Reynolds number, Re'. Macdonald *et al.* [2] proposed a fixed value of 180 for A, while suggesting that B varied from a value of 1.8 for smooth particles to 4.0 for rough particles.

There are other Ergun based pressure drop correlations in the literature which mainly cast doubt on the universality of the constants 150 and 1.75 in the original equation (1). For example, Handley and Heggs [4] proposed a correlation having constants of 368 and 1.24, instead of the respective values of 150 and 1.75 as proposed by Ergun [1].

However, recent work on the characterisation of structure of packed beds [5, 6] has shown that the diameter ratio,

NOMENCLATURE

A	intercept in equation (3)
B	slope in equation (3)
d_p	particle diameter [m]
d_r	tube-to-particle diameter ratio
d_t	tube diameter [m]
f'	modified friction factor
L	tube length [m]
M	parameter in equation (4)
∇P	pressure gradient [Pa m^{-1}]
Re'	modified Reynolds number
u	superficial velocity [m]
V_0	superficial velocity [m].

Greek symbols

α	parameter in equation (1)
β	parameter in equation (1)
ε_m	mean voidage
μ	viscosity [N s m^{-2}]
ρ	density [kg m^{-3}].

d_t/d_p, is an important structural parameter in packed bed configurations, and that the mean voidage, ε_m, of packed beds is a function of this diameter ratio. Hence, because the mean voidage affects the pressure drop-flow properties of packed beds, it is expected that the diameter ratio, d_t/d_p, plays a role in any attempt to develop predictive pressure drop-flow correlations, especially in the range where the so-called "wall effect" is particularly noticeable.

One of the earlier attempts to include the effect of confining walls on the pressure drop characteristics of packed beds was that of Mehta and Hawley [7]. They argued that since the development of the Ergun equation (1) involves the hydraulic radius as a characteristic length, then the wall must be taken into account for small tube to particle diameter ratios. For large diameter ratios, however, the wall should have little effect on the characteristic length and can therefore be ignored. On the basis of this argument they proposed the following modified correlation:

$$\frac{\Delta P}{L}\frac{d_p\varepsilon^3}{\rho u^2(1-\varepsilon)}\frac{1}{M} = \frac{150\mu(1-\varepsilon)M}{d_p\rho u}+1.75 \qquad (4)$$

where M is a parameter defined as

$$M = 1+\frac{4d_p}{6d_t(1-\varepsilon)}$$

and is used to correct the hydraulic radius R_h of a packed bed for the effect of wall

$$R_h = \frac{\varepsilon d_p}{6(1-\varepsilon)M}.$$

Equation (4) reduces to Ergun equation (2) when $M \to 1$ (i.e. when $d_t/d_p \gg 1$). Recently Paterson *et al.* [8] attempted to refine the pressure drop correlation by incorporating a diameter ratio term, d_t/d_p. Unfortunately, lack of sufficient information on the derivation of the empirical equation does not allow a critical assessment of the correlation to be made.

The Ergun-type pressure drop correlations include voidage terms raised to the power 3 and so it is important that accurate values of voidage are used in any calculation. A common source of error is the assumption that mean voidage of packed beds of spherical particles is approximately equal to 0.4. While this may be acceptable for beds with relatively large tube to particle diameter ratios, it is certainly not realistic for low diameter ratios, i.e. $d_t/d_p < 10.0$.

Recent work provided a predictive correlation for mean voidage in packed beds of spherical particles [6, 9]

$$\varepsilon_m = 1-\frac{2}{3}\left(\frac{1}{d_r}\right)^3\frac{1}{\sqrt{\left(\frac{2}{d_r}-1\right)}} \qquad (5)$$

$$\text{for } 1 \leqslant d_r \leqslant 1+\frac{\sqrt{3}}{2}$$

$$\varepsilon_m = 0.383+0.25d_r^{-0.923}\frac{1}{\sqrt{(0.723d_r-1)}} \qquad (6)$$

for $1+\sqrt{3}/2 < d_r$ where $d_r = d_t/d_p$.

For equation (6), the sum of squares of errors and mean square errors were respectively 23.9 and 1.96 [6], showing that reliable mean voidage values can be predicted.

The purpose of this work is to examine the effect of various contributory factors, such as structural properties, on the pressure drop of fluids through packed beds so that appropriate correlations with wide practical applicability can be established.

THEORETICAL CONSIDERATION

The effect of the confining wall on the channelling of flow is said to be important when the diameter ratio, d_t/d_p, is less than about 50 [10], and becomes more pronounced at values less than 12. This dependency has been confirmed by recent work on the structure of packed beds using techniques such as Image Analysis [5, 6, 11]. The presence of boundary walls causes a less efficient packing of particles, and hence enhanced voidage, in the wall region of the bed and as a consequence the flow of fluids in the wall zone will be significantly increased [12]. The trends in the flow and structural characteristics of particulate beds signify the importance of taking into account the physical dimensionality of the system when investigating the pressure drop-flow properties.

Therefore, if pressure drop-flow experiments were carried out in packed beds of different diameter ratios, d_t/d_p, then a plot of the modified friction factor vs the modified Reynolds number, equation (3), should be linear. Since the diameter ratio is the key parameter in these considerations, analysis of data extracted from linear regression procedures should provide an indication of any possible functional relationship between the diameter ratio and the regression coefficients.

In order to test the above theoretical considerations, a series of experiments have been conducted using a range of physical and operating conditions, $3 < d_t/d_p < 25$ and $5 < Re' < 8500$, with a view to be able to examine the effects of factors, such as containing walls and system dimensionality, on the pressure drop of the flowing fluid across the packing matrix.

EXPERIMENTAL WORK

The pressure drop experiments have been carried out using a rig consisting of three flowmeters arranged in parallel, preceded by a pressure gauge, a pressure regulator and a filter to retain any moisture or oil from the air compressor. The packed column consisted of a Perspex tube of 50 mm i.d., filled with spherical particles of desired size to a height of 30 cm. The air enters the column from the bottom and flows through a piece of knitmesh in order to be uniformly distributed. The bed is provided with two pressure probes, one at the inlet of the packing and the other immediately

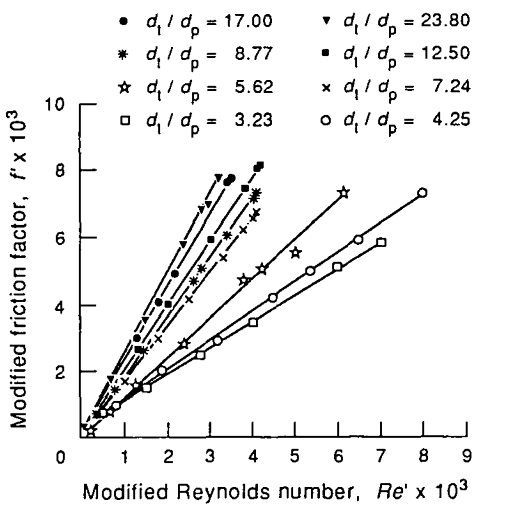

FIG. 1. Plot of modified friction factor, f', vs modified Reynolds number, Re', using experimental data.

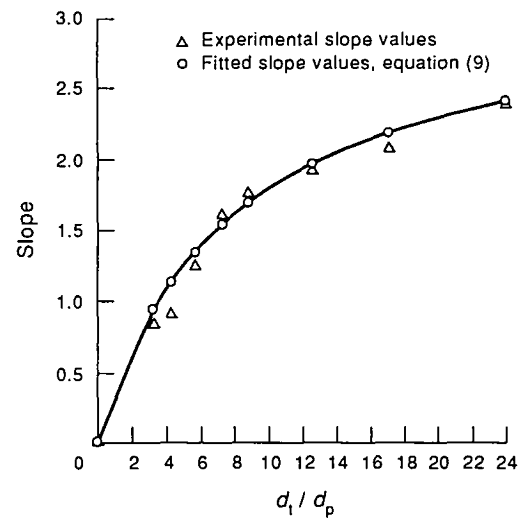

FIG. 2. Variation of the slope with diameter ratio, d_t/d_p.

after the packing, and the pressure drop is read from a paraffin or mercury manometer. The particulate column is formed by slow feeding of the spherical particles of desired size into a cylindrical container. The bed is subsequently subjected to a gentle vibration in order to ensure a firm bed. This procedure has been adopted for each of the experiments.

RESULTS AND DISCUSSION

A series of experiments have been conducted in order to obtain pressure drop-flow data for a number of packed beds. The relationship between the modified Reynolds number, Re', and the modified friction factor, f', equation (3), has been studied by a linear regression analysis. Figure 1 depicts such plots. The mean voidage of each bed, ε_m, is obtained from equation (6). Inspection of the pressure drop data, Fig. 1, reveals that while the slope, i.e. coefficient B in equation (3), increases as the diameter ratio increases, the intercept, A in equation (3), varies randomly over a rather narrow range of values as compared with the full range in the y-axis. These results signify some form of direct proportionality between the slope and the diameter ratio. Figure 2, however, shows that the slope does not increase linearly with the diameter ratio, d_t/d_p, but rather tends to a limiting value as d_t/d_p increases. On the other hand, the intercept does not appear to follow any noticeable trend, and the cause of its random variation is believed to be due to the experimental and fitting errors which tend to be more pronounced at extreme operating conditions. On the basis of this, the value of the intercept is taken as a constant and independent of the physical dimensionality of the system. Statistical analysis of the experimental data can provide the best estimate of its value. For this, the observed data of all the beds have been treated to facilitate the optimized value of the intercept which is found to be 130. The corresponding slopes of the eight beds are shown graphically in Fig. 2, where the values of the slope, B, are plotted against the diameter ratio, d_t/d_p.

From Fig. 2, it can be seen that the relationship between B and d_t/d_p is of a non-linear nature and for this the Marquardt optimization routine [13], in conjunction with a number of mathematical expressions, has been used to provide the best functional description of the B vs d_t/d_p curve. The following relationship is found to give the best fit with a standard error of estimate of 0.23:

$$B = \frac{d_t/d_p}{0.335 d_t/d_p + 2.28}. \qquad (7)$$

Substituting for B and A into equation (3) provides the generalized pressure drop correlation

$$f' = \left(\frac{(d_t/d_p)}{0.335(d_t/d_p) + 2.28}\right) Re' + 130. \qquad (8)$$

Figure 3 shows the graphical representation of the modified friction factor, f', vs the modified Reynolds number, Re', for several diameter ratio values.

Having established the necessary pressure drop correlation incorporating the appropriate term for the dimensionality of the physical system, it is desirable to compare it with existing correlations. For this purpose, correlations proposed by Ergun [1], Handley and Heggs [4], Macdonald *et al.* [2] and Mehta and Hawley [7] have been considered. Figures 4–6 show comparative information for three beds with diameter ratios of 3.5, 7.5 and 15.0, respectively. The voidage values used in all cases are obtained from equation (6). Inspection of the results reveals interesting features. For a bed with relatively large diameter ratio, $d_t/d_p = 15.0$, all the rival pressure drop curves reasonably coincide with each other, but as the diameter ratio decreases, the discrepancies between the rival predictions tend to become more significant. The interesting feature of the graphically presented results is the trend between different predictions. For a set of given conditions, the existing correlations, in general, tend to over-estimate the pressure drop of fluids through the particulate beds. This can be easily explained by the fact that the existing correlations were derived from experiments performed at high d_t/d_p values while the present work covers a wide range of diameter ratios. This clearly signifies the limited applicability of the existing correlations. This finding is quite significant, since most industrial multitubular catalytic reactors have, in fact, low diameter ratios, < 15, and hence an accurate prediction of pressure drop is highly desirable. It is also surprising to notice that although Mehta and Hawley's correlation [7] is supposed to take into account the confining wall, the pressure drops predicted are overestimated by over 100% at d_t/d_p equal to 3.50.

CONCLUSION

A modified pressure drop correlation has been established which accounts for the presence of the boundary walls. The

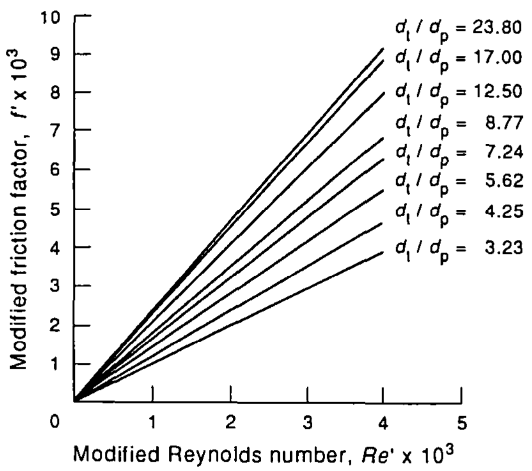

FIG. 3. Plot of modified friction factor, f', vs modified Reynolds number, Re', according to equation (10).

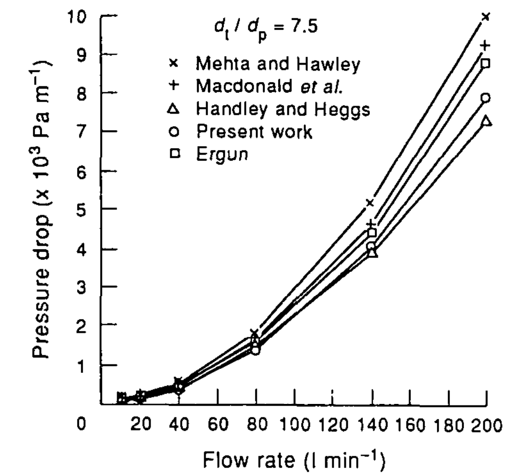

FIG. 5. Comparison of pressure drops obtained in the present work and selected literature correlations at $d_t/d_p = 7.5$.

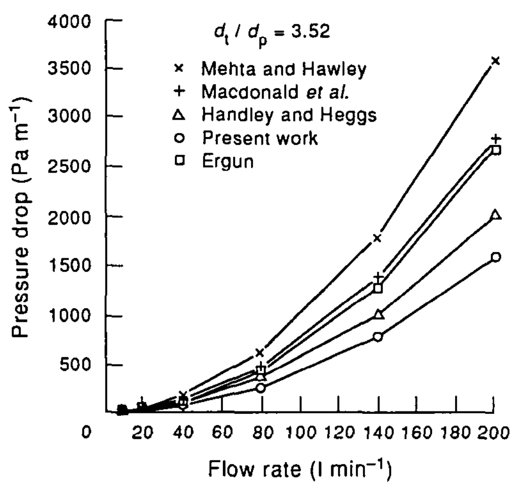

FIG. 4. Comparison of pressure drops obtained in the present work and selected literature correlations at $d_t/d_p = 3.52$.

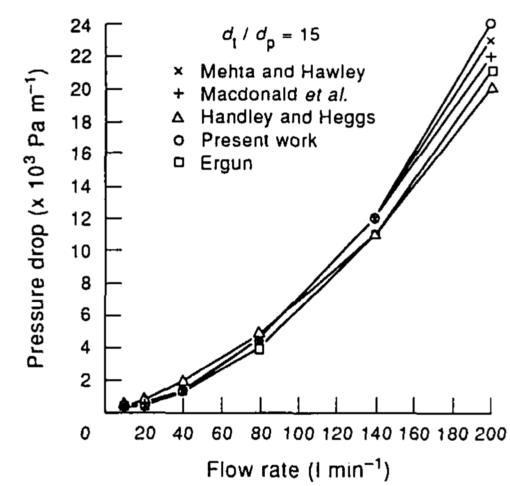

FIG. 6. Comparison of pressure drops obtained in the present work and selected literature correlations at $d_t/d_p = 15$.

relationship provides reliable information on the pressure drop across the matrix. Analysis of the observed data demonstrated the significance of the dimensionality parameter, namely d_t/d_p, on the pressure drop. A comparison between the present and the existing correlations showed that the published correlations cannot be expected to facilitate reliable pressure drop, particularly for packed beds with $d_t/d_p \leqslant 10$. This is explained by the fact that the existing pressure drop correlations were derived from experiments carried out with packed beds of relatively large diameter ratios, and hence do not account adequately for the strong wall effect which is known to exist in low diameter ratio beds. It is, however, in this low end of the diameter ratios that most multitubular catalytic reactors and single shot packed beds fall, and the existing pressure drop correlations fail to facilitate dependable design information. The proposed correlation (equation (10)) is believed to contribute significantly to the design and performance prediction of packed beds of spherical particles.

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